

Bohmian picture of quantum polarization: Nonclassical behavior of classical-like states

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We address a trajectory picture of quantum polarization by applying the tools of the Bohmian representation of quantum mechanics to some relevant examples of two-mode field states, such as Glauber and SU(2) coherent states. We show that the corresponding electric-field trajectories are incompatible with classical electrodynamics, despite the fact that these states are often considered as classical regarding polarization properties.

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INTRODUCTION

Polarization is a distinguished laboratory for the analysis and application of fundamental quantum ideas. Specifically, in this contribution we address a trajectory picture of quantum polarization by applying the tools of the Bohmian representation of quantum mechanics [1].

According to its standard definition, polarization refers to the ellipse described in time by the real electric-field vector of harmonic waves. Hence partial polarization can be understood as the rapid and random succession of more or less different polarization states. When moving to the quantum domain we find that the electric field can never describe a perfect ellipse, just in the same way that particles cannot follow definite trajectories [2]. This is because the (field) quadratures of a single-mode field essentially satisfy the same commutation relations of position and linear momentum.

However, suitable well-defined trajectories can be introduced in quantum mechanics via its Bohmian formulation. In a previous work [3] we have obtained the trajectories described by the electric field of one-photon two-mode states. In this contribution we extend the analysis to some relevant examples of two-mode field states, such as Glauber and two-photon SU(2) coherent states [4, 5]. We show that the corresponding electric-field trajectories are incompatible with classical electrodynamics, despite the fact that these states are often considered as classical regarding polarization properties.

POLARIZATION BOHMIAN TRAJECTORIES

Bohmian trajectories described by a two-mode electric field are given by solving the guidance equation $\dot{\mathbf{E}} = \nabla S$ where $\mathbf{E} = (E_x, E_y)$ are the transversal real electric-field strengths along Cartesian axes, S is the phase of the field-state wave function in quadrature representation, and the gradient ∇S is taken with respect to $\mathbf{E} = (E_x, E_y)$. Since polarization is the evolution of the electric field,

the configuration space for polarization is given by the electric-field variables \mathbf{E} , which play the same role of the coordinates \mathbf{r} for the dynamics of a particle. Throughout we will consider units in which field frequency and reduced Planck constant are unity. Next we solve the guidance equation for some relevant classical-like quantum field states.

Glauber coherent states

For Glauber coherent states $|\alpha_x, \alpha_y\rangle$ the quadrature wave function is Gaussian,

$$\psi(\mathbf{E}, t) \propto e^{i(\langle \tilde{E}_x \rangle_t E_x + \langle \tilde{E}_y \rangle_t E_y)} \times e^{-(E_x - \langle E_x \rangle_t)^2 / 2 - (E_y - \langle E_y \rangle_t)^2 / 2}, \quad (1)$$

where $\langle E_j \rangle_t$ and $\langle \tilde{E}_j \rangle_t$, $j = x, y$, are real quantities defined as $\sqrt{2}\alpha_j e^{-it} = \langle E_j \rangle_t + i\langle \tilde{E}_j \rangle_t$. The guidance equation can be exactly solved to give

$$E_j(t) = E_j(0) + \sqrt{2}|\alpha_j| \cos(t - \delta_j) - \sqrt{2}|\alpha_j| \cos \delta_j, \quad (2)$$

where $\delta_j = \arg \alpha_j$. In Fig. 1 we have represented the contour plots of the one-cycle averaged field distribution

$$\bar{P}(\mathbf{E}) \propto \int_0^{2\pi} |\psi(\mathbf{E}, t)|^2 dt, \quad (3)$$

and three examples of the trajectories described by Eq. (2). The solid line is the most probable trajectory starting at the maximum of (1) at $t = 0$, this is $E_j(0) = \langle E_j \rangle_{t=0} = \sqrt{2}|\alpha_j| \cos \delta_j$. The other trajectories start from points at half variance from the maximum. We can appreciate that all trajectories have exactly the same form, which coincides with the polarization ellipse of a classical harmonic wave with complex amplitudes $\alpha_{x,y}$. However, only the most probable is centered at the origin, while all the others are slightly displaced. This displacement is in contradiction with classical electrodynamics, despite coherent states are typical examples of classical-like light.

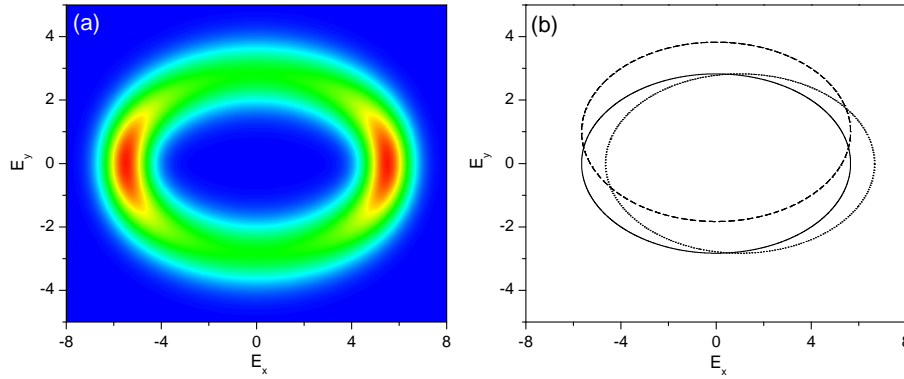


FIG. 1: (a) Contour plots of the one-cycle averaged field distribution $\overline{P}(\mathbf{E})$ for a two-mode coherent state with $\alpha_x = 4$, $\alpha_y = 2i$. (b) Three Bohmian trajectories obtained from Eq. (2) for different initial conditions.

One-photon SU(2) coherent states

The constraint of Glauber coherent states to subspaces with a fixed total photon number gives rise to the well-known coherent states of the SU(2) algebra [5]. They are often regarded as the most classical spin states [4, 5]. For completeness, we begin our analysis with the one-photon SU(2) coherent states already examined in Ref. [3]. We consider the same Stokes parameters as for the above Glauber coherent state, so that the quadrature wave function is

$$\psi(\mathbf{E}, t) \propto (\alpha_x E_x + \alpha_y E_y) e^{-(E_x^2 + E_y^2)/2} e^{-it}. \quad (4)$$

In Fig. 2(a) we have represented the field strength distribution $P(\mathbf{E}) = |\psi(\mathbf{E}, t)|^2$, which is actually independent of time since the field state is stationary. In panel (b) we have plotted several electric-field Bohmian trajectories, which are all then circular and centered at the origin. Note that the origin is the only point where the wave function vanishes $\psi(\mathbf{E}, t) = 0$. As shown in Ref. [3], all one-photon states have circular trajectories, even though they provide us with full information about the mean polarization state that is contained in a time-dependent angular frequency.

Two-photon SU(2) coherent states

The Bohmian trajectories associated with two-photon SU(2) coherent states already display a more striking dynamical behavior than in the previous two cases. We again consider the same Stokes parameters of the above examples

$$\begin{aligned} \psi(\mathbf{E}, t) &\propto e^{-(E_x^2 + E_y^2)/2} e^{-2it} \\ &\times [\alpha_x^2 (2E_x^2 - 1) + \alpha_y^2 (2E_y^2 - 1) + 4\alpha_x \alpha_y E_x E_y]. \end{aligned} \quad (5)$$

In Fig. 3(a) we have represented a contour plot of the field strength distribution $P(\mathbf{E}) = |\psi(\mathbf{E}, t)|^2$, which is

again independent of time. In panel (b), we have plotted several electric-field Bohmian trajectories. The solid line corresponds to approximately the most probable trajectory, which starts at the right-hand side maximum, at $\mathbf{E} = (1.5, 0)$. The other two trajectories start at $\mathbf{E} = (0.4, 0)$ (dotted line) and $\mathbf{E} = (0, 0.001)$ (dashed line). Note that while the most probable resembles the classical polarization ellipse, the other two trajectories are in sharp contradiction with the predictions of classical electrodynamics concerning the evolution of the electric field \mathbf{E} . These trajectories seem to orbit the two only points where the electric-field wave function vanishes: $\psi(\mathbf{E}, t) = 0$ at $E_y = 0$, $E_x = \pm\sqrt{3/8}$.

CONCLUSIONS

We have addressed a Bohmian approach to light polarization in quantum optics by computing the trajectories described by the electric field of some classical-like two-mode states. For Glauber coherent states all trajectories have the same elliptic form of the mean field, but are not centered at the origin. For SU(2) coherent states trajectories are even far from being ellipses. These results are in contradiction to classical electrodynamics. This is quite remarkable since these field states are universally regarded as classical-like concerning polarization. In this regard it is worth noticing that the definition of the electric-field Bohmian trajectories has no straightforward classical counterpart, since there seems to be no simple classical version of the phase of the electric-field wave function.

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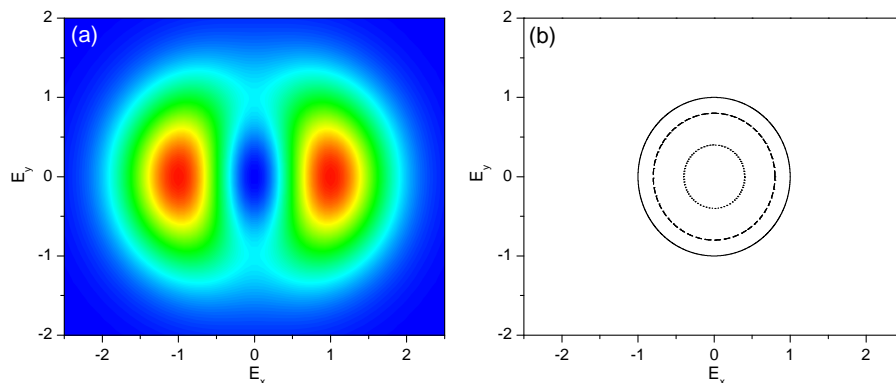


FIG. 2: (a) Contour plot of the stationary field distribution $P(\mathbf{E})$ for a one-photon SU(2) coherent state with the same Stokes parameters as the Glauber coherent state (1). (b) Three Bohmian trajectories for the same field state and different initial conditions.

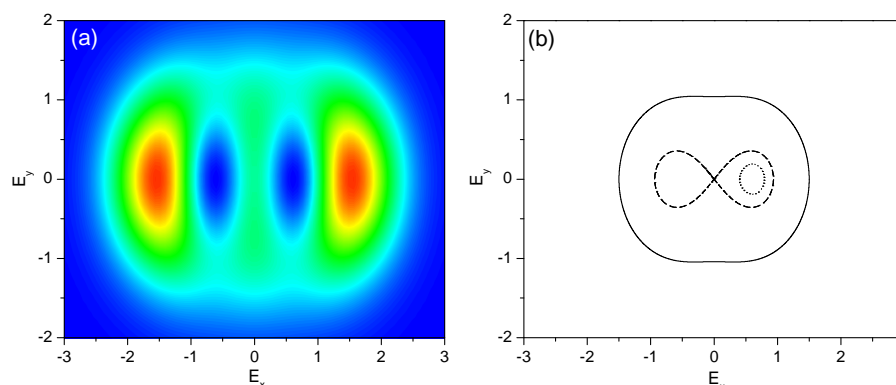


FIG. 3: (a) Contour plot of the stationary field distribution $P(\mathbf{E})$ for a two-photon SU(2) coherent state with the same Stokes parameters used in the previous examples. (b) Three Bohmian trajectories for the same field state and different initial conditions.

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